Answer Key
AP Calculus AB Pre-Exam

The Calculus Pre-Exam is a tool for evaluating whether a student is ready to take the AP Calculus AB Course offered by Patrick Henry College Preparatory Academy. After taking the exam, students or their parents may download the Pre-Exam Answer Key to grade the exam. A student should be able to score at least 80% on the exam before signing up for the course.

This test covers (1) basic algebraic skills, (2) basic skills involving functions, and (3) basic trigonometric skills. A student should be able to effectively use mathematical skills in all three of these areas before taking AP Calculus AB.

For each of the following questions, choose the best response.

Problems (1) – (10) test basic algebraic skills:

1. If \( n \) and \( m \) are integers and \( a \) and \( b \) are real numbers, which of the following statements is incorrect?
   (A) \( a^n \cdot a^m = a^{n+m} \)
   (B) \( a^n \cdot b^{-m} = a^n \div b^m \)
   (C) \( a^{nm} = (a^m)^n \)
   (D) \( a^n \div a^{-m} = a^{n-m} \) Correct answer
   (E) \( (ab)^{nm} = a^{nm} \cdot b^{nm} \)

2. The expression \( \frac{1-x-12x^2}{1-9x^2} \) can be simplified to which of the following choices?
   (A) \( \frac{1-4x}{1+3x} \)
   (B) \( \frac{1-3x}{1+2x} \)
   (C) \( \frac{4x-1}{2x-1} \)
   (D) \( \frac{1-4x}{1-3x} \) Correct answer
   (E) \( \frac{1-x}{1+6x} \)
3. \[ \frac{a+2}{2a-6} - \frac{a-2}{2a+6} = \text{which of the following choices?} \]
   
   (A) \( \frac{9a}{2(a^2-9)} \)
   
   (B) \( \frac{1}{a} \)
   
   (C) \( \frac{a^2+6}{a^2-9} \)
   
   (D) \( \frac{5a}{a^2-9} \) \text{ Correct answer}
   
   (E) \( \frac{a+2}{4a} \)

4. Which of the following is a correct factoring of the expression \( x^4 + 24x^2y^2 - 25y^4 \)?

   (A) \( (x - 5y)(x + 5y)(x^2 + y^2) \)
   
   (B) \( (x - y)(x + y)(x^2 + 25y^2) \) \text{ Correct answer}
   
   (C) \( (x - 4y)(x + 6y)(x^2 + 5y^2) \)
   
   (D) \( (x - 6y)(x + 4y)(x^2 - y^2) \)
   
   (E) \( (x - y)(x + 5y)(x^2 + 5y^2) \)

5. If \( y = 4x^2 - 5x + 4 \), what is the value of \( y \) when \( x = 2 \)?

   (A) 2
   
   (B) 5
   
   (C) 12
   
   (D) 3
   
   (E) 10 \text{ Correct answer}

6. If \( y = \frac{x-2}{(x-3)(x+4)} \), \( y \) cannot be evaluated for what value of \( x \)?

   (A) 2
   
   (B) -3
   
   (C) 4
   
   (D) -4 \text{ Correct answer}
   
   (E) 0
7. What are the values of $x$ for which the following is it true: $(x + 2)(x^2 - 1) = 0$?

(A) $x = 2, -1, and 1$

(B) $x = -2, 1, and -1$  Correct answer

(C) $x = -1 and 2$

(D) $x = 1 and 2$

(E) $x = 0, 1, and 2$

8. The solution of the equation $3x - 2 = 5 - \frac{x}{2}$ is which of the following?

(A) $x = 2$

(B) $x = \frac{1}{2}$

(C) $x = 1$

(D) $x = -1$

(E) $x = 4$  Correct answer

9. One number is 5 more than another and the sum of the two numbers is 25. What is the smaller of the two numbers?

(A) 5

(B) 10  Correct answer

(C) 15

(D) 8

(E) 12

10. Let $w = -2x + 4$. For what value or values of $x$ is $w > 0$?

(A) $x = 4$

(B) $x > 2$

(C) $x < 2$  Correct answer

(D) $x > 0$

(E) $x < 0$
Problems (11) – (20) test basic skills involving the concept of functions:

11. Suppose \( h(x) = x + 1 \) and \( g(h) = h^2 - 3 \). What is \( g \) as a function of \( x \)?
   (A) \( g(x) = 2x - 1 \)
   (B) \( g(x) = x - 2 \)
   (C) \( g(x) = x^2 - 3 \)
   (D) \( g(x) = x^2 + 2x - 2 \)  \( \text{Correct answer} \)
   (E) \( g(x) = x^2 + 2x + 1 \)

12. Suppose \( f(x) = x^2 \) and \( h(x) = x \). Which of the following is not true?
   (A) \( f(x) > h(x) \) when \( x > 1 \)
   (B) \( f(x) > h(x) \) when \( x \geq 0 \)  \( \text{Correct answer} \)
   (C) \( f(x) > h(x) \) when \( x < 0 \)
   (D) \( f(x) = h^2(x) \)
   (E) \( h(x) = \sqrt{f(x)} \) when \( x > 1 \)

13. Suppose \( f(x) = x^{-1} \) and \( h(x) = x \). Which of the following is not true?
   (A) \( f(x) \) is not defined at \( x = 0 \).
   (B) \( f(x) > h(x) \) when \( x < -1 \)
   (C) \( f(x) < h(x) \) when \( x > 1 \)
   (D) \( f(0) = h(0) \)  \( \text{Correct answer} \)
   (E) \( h(x) > \sqrt{f(x)} \) when \( x > 1 \)

14. Suppose \( f(x) = 2x \) and \( h(x) = 1 \). For what \( x \) value is it true that \( f(x) - h(x) = 0 \)?
   (A) \( x = 2 \)
   (B) \( x = 0.5 \)  \( \text{Correct answer} \)
   (C) \( x = -1 \)
   (D) \( x = 1 \)
   (E) \( x = -0.5 \)
15. Suppose \( f(x) = x^3 \) and \( h(x) = x \). For what \( x \) values is it true that \( f(x) = h(x) \)?

(A) \( x > 0 \)
(B) \( x < 0 \)
(C) \( x = 0 \) and \( x = \pm 1 \) \( \text{Correct answer} \)
(D) \( x = 3 \) and \( x = \pm 1 \)
(E) no \( x \) values satisfy the condition

16. Consider the function \( f(x) = x^2 - 4 \). For what \( x \) is it true that \( f(x) \geq 0 \)?

(A) \( x \leq -2 \) and \( x \geq 2 \) \( \text{Correct answer} \)
(B) \( x \geq -2 \) and \( x \leq 2 \)
(C) all values of \( x \)
(D) no values of \( x \)
(E) \( x = \pm 2 \)

17. Let \( f(x) = \frac{x^2}{(x-3)(x+4)} \); \( f(x) \) is not defined for which of the following values of \( x \)?

(A) 2
(B) 3 \( \text{Correct answer} \)
(C) 4
(D) −3
(E) 0

18. Suppose \( f(x) = (x + 2)(x^2 - 1) \). For which of the following \( x \) values is it true that \( f(x) < 0 \)?

(A) \( x < -1 \)
(B) \( x > 1 \)
(C) \( x > -1 \) and \( x < -1 \)
(D) \( x > -1 \) and \( x < 1 \) \( \text{Correct answer} \)
(E) no values of \( x \)
19. Suppose \( f(x) = \frac{3x - 2}{5} \) and \( (x) = 4 - \frac{x}{2} \). For what \( x \) value do these functions map into the same value?

(A) \( x = 2 \)
(B) \( x = \frac{1}{2} \)
(C) \( x = 1 \)
(D) \( x = -1 \)
(E) \( x = 4 \)  
Correct answer

20. A rectangle has a height \( h \), width \( w \), and a perimeter of length 30. Express \( h \) as a function of \( w \).

(A) \( h(w) = \frac{30}{w} \)
(B) \( h(w) = 15 - w \)  
Correct answer
(C) \( h(w) = 30 - 2w \)
(D) \( h(w) = 30 - w \)
(E) \( h(w) = \sqrt{30 - w^2} \)

Problems (21) – (30) test basic skills involving trigonometry:

Problems 21, 22, and 23 refer to the following figure showing triangle \( ABC \) with angles \( a \), \( b \), and \( c \); sides \( AB \), \( BC \), and \( AC \); and the line \( BD \) which is perpendicular to side \( AC \). We will indicate length of lines with a pair of vertical lines. Thus, \( |AB| \) is the length of side \( AB \).

21. Which of the following is \( \sin(a) \)?

(A) \( \sin(a) = |BC| \div |AC| \)
(B) \( \sin(a) = |BC| \div |AB| \)
(C) \( \sin(a) = |BD| \div |AB| \)  
Correct answer
(D) \( \sin(a) = |BC| \cdot |AC| \)
(E) \( \sin(a) = |BD| \cdot |AC| \)
22. Which of the following is \( \cos(c) \)?

(A) \( \cos(c) = |CD| \div |BC| \) **Correct answer**  
(B) \( \cos(c) = |BC| \div |AB| \)  
(C) \( \cos(c) = |BD| \div |BC| \)  
(D) \( \cos(c) = |BC| \cdot |AC| \)  
(E) \( \cos(c) = |BD| \cdot |AC| \)

23. Which of the following is \( \tan(a) \)?

(A) \( \tan(a) = |BC| \div |AC| \)  
(B) \( \tan(a) = |BC| \div |AB| \)  
(C) \( \tan(a) = |BD| \div |AB| \)  
(D) \( \tan(a) = |BD| \div |AD| \) **Correct answer**  
(E) \( \tan(a) = |BD| \cdot |AC| \)

24. For which angle \( \theta \) is \( \tan(\theta) \) not defined?

(A) \( \theta = 0^\circ \)  
(B) \( \theta = 180^\circ \)  
(C) \( \theta = 90^\circ \) **Correct answer**  
(D) \( \theta = 360^\circ \)  
(E) \( \tan(\theta) \) is defined for all \( \theta \).

25. Let \( \theta \) be some angle for which \( \cos(\theta) = \sin(45^\circ) \). For which of the following \( \theta \) values is this true?

(A) \( \theta = 0^\circ \)  
(B) \( \theta = 135^\circ \)  
(C) \( \theta = -45^\circ \) **Correct answer**  
(D) \( \theta = 225^\circ \)  
(E) none of the above.
26. It is known that \( \tan(45°) = 1 \). For which of the following \( \theta \) values is it also true that \( \tan(\theta) = 1 \)?
   (A) \( \theta = 0° \)
   (B) \( \theta = 135° \)
   (C) \( \theta = -45° \)
   (D) \( \theta = 225° \)  \( \text{Correct answer} \)
   (E) none of the above.

27. What is the minimum value of \( \cos(\theta + 45°) \)?
   (A) 0
   (B) \( \frac{1}{\sqrt{2}} \)
   (C) 1
   (D) \( -1 \)  \( \text{Correct answer} \)
   (E) \( -\frac{1}{\sqrt{2}} \)

28. Which of the following is not true?
   (A) \( \tan(\theta) = \sin(\theta) \div \cos(\theta) \)
   (B) \( \sin^2(\theta) + \cos^2(\theta) = 1 \)
   (C) \( \cos(\theta) = \sin(90° - \theta) \)
   (D) \( \cos(\theta + 90°) = -\sin(\theta) \)
   (E) \( \cos(-\theta) = -\cos(\theta) \)  \( \text{Correct answer} \)

29. Suppose \( f(x) = \cos(x) \) and \( n \) is an integer. Which of the following is true?
   (A) \( f(x + n \cdot 90°) = f(x) \)
   (B) \( f(x + n \cdot 360°) = f(x) \)  \( \text{Correct answer} \)
   (C) \( f(x + n \cdot 100°) = f(x) \)
   (D) \( f(x + n \cdot 180°) = f(x) \)
   (E) \( f(x + n \cdot 200°) = f(x) \)
30. Consider the function \( \cos(3\theta) \). For which of the following \( \theta \) values is this function equal to 0?

(A) \( \theta = 0^\circ \)

(B) \( \theta = 60^\circ \)

(C) \( \theta = 30^\circ \)  \textit{Correct answer}

(D) \( \theta = 45^\circ \)

(E) none of the above